**ME 201/APh 250, Homework 3:**

Assigned: Friday, Apr 19, 2019
Due: Friday, Apr 26, 2019

**N&C 5.1**:
We can define the quantum Fourier transform as

$$F = \frac{1}{\sqrt{N}} \sum_{m,n=0}^{N-1} e^{2\pi imn/N} |m\rangle \langle n|$$

Note that $F|j\rangle = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ijk/N} |m\rangle \langle n|$ by construction. Then,

$$\begin{aligned}
FF^\dagger &= (\frac{1}{\sqrt{N}} \sum_{m,n=0}^{N-1} e^{2\pi imn/N} |m\rangle \langle n|)(\frac{1}{\sqrt{N}} \sum_{r,s=0}^{N-1} e^{-2\pi irs/N} |r\rangle \langle s|) \\
&= \frac{1}{N} \sum_{m,n,r,s=0}^{N-1} e^{2\pi imn/N} e^{-2\pi irs/N} |m\rangle \langle s| \delta_{n,r} \\
&= \frac{1}{N} \sum_{m,n,s=0}^{N-1} e^{2\pi imn/N} e^{-2\pi ins/N} |m\rangle \langle s| \\
&= \sum_{m,s}^{N-1} |m\rangle \langle s| (\frac{1}{N} \sum_{n=0}^{N-1} e^{2\pi in(m-s)/N}) \\
&= \sum_{m,s}^{N-1} |m\rangle \langle s| \delta_{m,s} \\
&= \sum_{m}^{N-1} |m\rangle \langle m| \\
&= I
\end{aligned}$$

By similar arguments, $F^\dagger F = I$. Therefore, the quantum Fourier transform is a unitary operator.

**N&C 5.2**:

Note that $|00...0\rangle \equiv |0\rangle$.

$$\begin{aligned}
F|0\rangle &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{2\pi ij0/N} |k\rangle \\
&= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} |k\rangle \\
&= (H^{\otimes n}) |0\rangle
\end{aligned}$$

**Qns 2**:

From phase estimation, we know that the eigenvalue $u$ can be estimated as $e^{2\pi i 0.\psi_1\psi_2\psi_3\psi 4}$ using a 4 qubit register for the phase estimation circuit. Consider the unitary $X$. We know the eigenvalues and eigenvectors are given by $1, |+\rangle$ and $-1, |-\rangle$. Let us assume we prepare the latter eigenvector. Then since $-1 = e^{2\pi i \frac{1}{2}}$, we expect the phase estimation circuit to give the binary form $0.1000 = 1 \times \frac{1}{2} + 0 \times \frac{1}{4} + 0 \times \frac{1}{8} + 0 \times \frac{1}{16}$. Indeed, running the circuit, this is what we get as shown in the figure below.

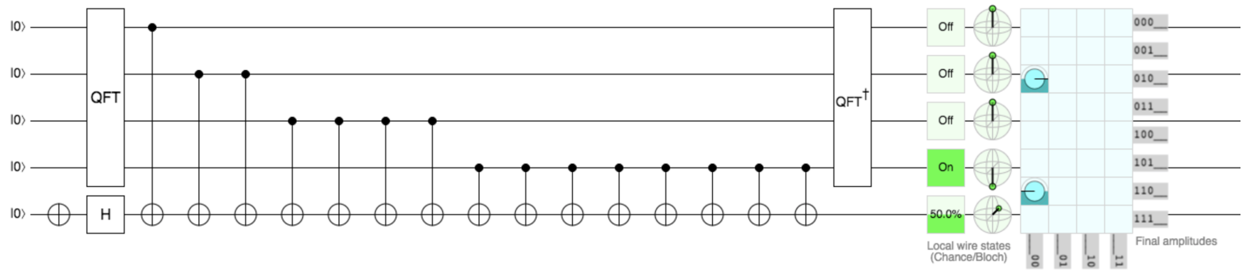


Figure 1: Estimating the eigenvalue that $|+\rangle$ correspond to. Note that the first index is the ancillary index and is unimportant.